Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel International GCSE

Centre Number | Candidate Number

Monday 20 January 2020

Morning (Time: 2 hours) | Paper Reference **4PM1/02**

Further Pure Mathematics

Level 2
Paper 2

Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page. Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶

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P 5 9 9 3 9 A 0 1 3 6

Pearson

International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere $= 4\pi r^2$

Curved surface area of cone $= \pi r \times$ slant height

Volume of sphere $= \frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

Geometric series

Sum to n terms, $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$

Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

Calculus

Quotient rule (differentiation)

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

Trigonometry

Cosine rule

In triangle ABC: $a^2 = b^2 + c^2 - 2bc\cos A$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$

$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$

Logarithms

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 A particle P is moving along a straight line that passes through the fixed point O. At time t seconds, $t \geqslant 0$, the displacement, s metres, of P from O is given by

$$s = t^3 + 4t^2 - 27t + 4$$

Find the value of t at the instant when the velocity of P is 8 m/s.

(4)

(Total for Question 1 is 4 marks)

2 Find the set of values of x for which

(a) $3 + 2x \leqslant x + 2$

(1)

(b) $8x^2 + 10x < 3$

(4)

(c) **both** $3 + 2x \leqslant x + 2$ **and** $8x^2 + 10x < 3$

(1)

Question 2 continued

(Total for Question 2 is 6 marks)

3

Diagram **NOT** accurately drawn

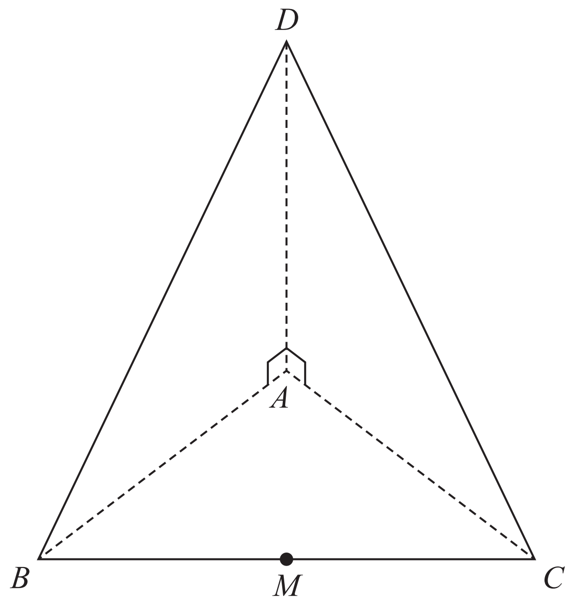

Figure 1

Figure 1 shows a triangular pyramid *ABCD*.

The base, *ABC*, of the pyramid is a horizontal isosceles triangle with $AB = AC = 10$ cm and $BC = 16$ cm. The midpoint of *BC* is *M*.

The face *BCD* of the pyramid is an isosceles triangle with $BD = CD = 26$ cm and *D* is vertically above *A*.

$\angle BAD = \angle CAD = 90°$

(a) Calculate the length, in cm, of *AM*.

(2)

Calculate, in degrees to the nearest degree,

(b) the size of $\angle BCD$,

(3)

(c) the size of the angle between the planes *BCA* and *BCD*.

(4)

Question 3 continued

Question 3 continued

Question 3 continued

(Total for Question 3 is 9 marks)

4 The points A, B, C and D are the vertices of a quadrilateral $ABCD$ such that

$$\overrightarrow{AB} = 7\mathbf{i} + p\mathbf{j} \qquad \overrightarrow{AC} = 11\mathbf{i} - p\mathbf{j} \qquad \overrightarrow{AD} = 4\mathbf{i} - 2p\mathbf{j}$$

(a) Show that, for all values of p, $ABCD$ is a parallelogram.

(3)

Given that $|\overrightarrow{BD}| = 3\sqrt{10}$

(b) find the possible values of p.

(3)

Given that $p > 0$

(c) find a unit vector which is parallel to $\overrightarrow{BD}$.

(1)

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Question 4 continued

Question 4 continued

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Question 4 continued

(Total for Question 4 is 7 marks)

5 Given that α and β are such that $\alpha + \beta = \dfrac{7}{2}$ and $\alpha\beta = 2$

(a) form a quadratic equation with integer coefficients that has roots α and β,

(2)

(b) form a quadratic equation with integer coefficients that has roots $\dfrac{\alpha}{\beta}$ and $\dfrac{\beta}{\alpha}$.

(6)

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Question 5 continued

(Total for Question 5 is 8 marks)

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6

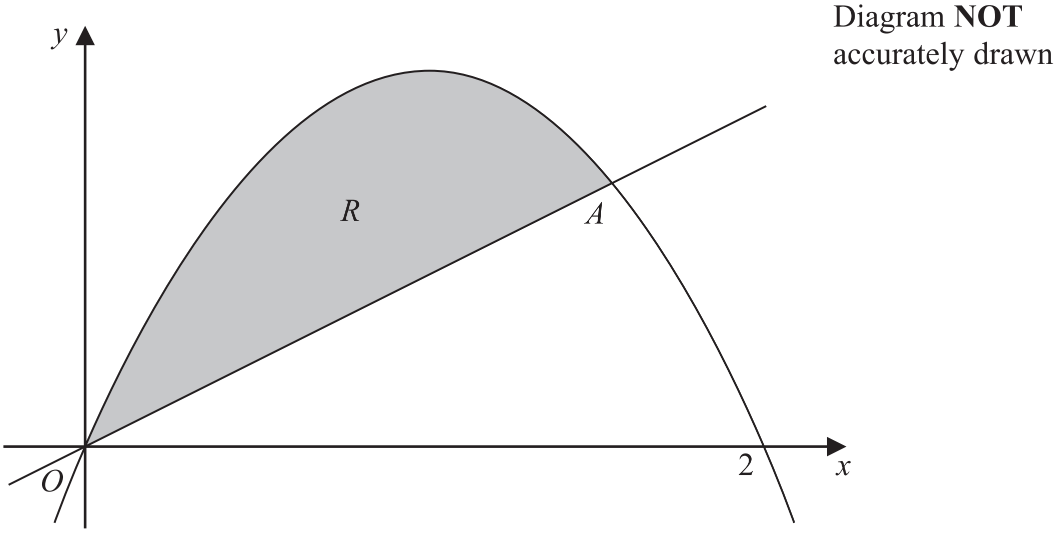

Figure 2

The region R, shown shaded in Figure 2, is bounded by the curve with equation $y = 2x - x^2$ and the line with equation $2y - x = 0$

The curve and the line intersect at the origin O and the point A.

(a) Show that the point A has coordinates $\left(\frac{3}{2}, \frac{3}{4}\right)$.

(2)

The region R is rotated through 360° about the x-axis.

(b) Use algebraic integration to find, in terms of π, the volume of the solid formed.

(6)

Question 6 continued

Question 6 continued

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Question 6 continued

(Total for Question 6 is 8 marks)

Turn over ▶

7 The 7th term of a geometric series is 192 and the 8th term of this geometric series is 1152

(a) Find, as a fraction in its simplest form, the 4th term of this geometric series.

(3)

A different geometric series G has a common ratio r and nth term t_n

Given that $t_3 = 24$ and $t_2 + t_3 + t_4 = -36$

(b) show that r satisfies the equation

$$2r^2 + 5r + 2 = 0$$

(5)

Given further that G is convergent with sum to infinity S,

(c) find the value of S.

(4)

Question 7 continued

Question 7 continued

P 5 9 9 3 9 A 0 2 2 3 6

Question 7 continued

(Total for Question 7 is 12 marks)

Turn over

8 Given that $y = e^{3x} \sin 2x$

show that $\dfrac{d^2y}{dx^2} - 6\dfrac{dy}{dx} + 13y = 0$

(8)

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 8 marks)

9 A curve C has equation

$$y = \frac{qx - 2}{x - p} \qquad x \neq p$$

The curve crosses the y-axis at the point A.

The line l with equation $y = x + 2$ is the normal to C at A.

(a) (i) Show that $p = 1$

(ii) Find the value of q.

(7)

(b) Using the axes on the opposite page, sketch C, showing clearly the asymptotes and the coordinates of the points where C crosses the coordinate axes.

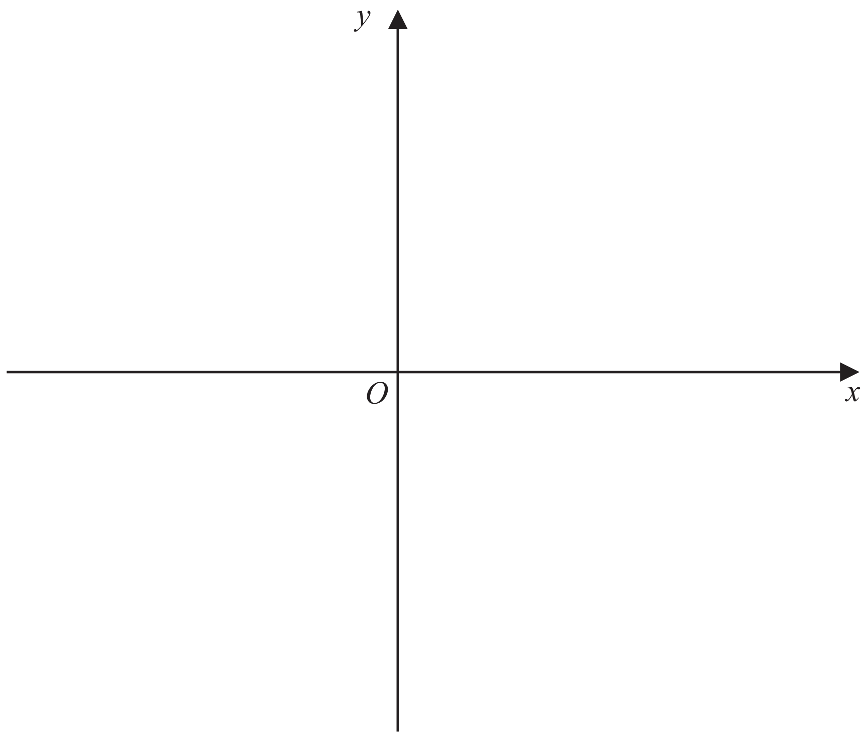
(5)

The line l meets C again at the point D.

(c) Find the x coordinate of D.

(4)

Question 9 continued



Question 9 continued

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Question 9 continued

(Total for Question 9 is 16 marks)

10 The volume of a sphere is increasing at a constant rate of 40 cm^3/s.

Find the rate of increase, in cm^2/s, of the surface area of the sphere at the instant when the radius is 4 cm.

(9)

Question 10 continued

(Total for Question 10 is 9 marks)

11 (a) Express the equation

$$3\sin(A - B) = \sin(A + B)$$

in the form $\tan A = k\tan B$, giving the value of the integer k.

(4)

(b) Given that $\theta \neq \dfrac{(2n+1)\pi}{2}$ where $n \in \mathbb{Z}$,

show that $\dfrac{\cos^4\theta - \sin^4\theta}{\cos^2\theta} = 1 - \tan^2\theta$

(3)

(c) Using the exact values of $\sin x°$, $\cos x°$ and $\tan x°$ for $x = 30, 45, 60$

show that

(i) $\cos 15° = \dfrac{\sqrt{6} + \sqrt{2}}{4}$

(2)

(ii) $\tan 255° = \dfrac{3 + \sqrt{3}}{3 - \sqrt{3}}$

(4)

Question 11 continued

Question 11 continued

(Total for Question 11 is 13 marks)

TOTAL FOR PAPER IS 100 MARKS